Use of Duty Cycles or Measured Torque—Time Data with AGMA Ratings

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Introduction

Variable loads resulting from a working process, starting process, or operation near a critical speed will cause varying stresses at the gear teeth of a drive system. The magnitude and frequency of these loads depend upon the driven machine, the motor, the dynamic mass elastic properties of the system, and other effects (Ref. 1).

These variable loads (stresses) may be determined by such procedures as:

- experimental measurement of the operating loads at the machine in question.
- calculation, using known external excitation and a mass elastic simulation of the drive system, preferably accompanied by experimental testing to validate the calculation.

In the automotive area torque and speed over time are usually measured on test rigs. Also, for industrial gearboxes, a measurement of torque over time is often used, for example in wind turbines. As measurement equipment and the transmission and storing of such data become less expensive, the tendency to measure torque/speed on gear drives is growing. Based on such data the service intervals can be adapted due to the analysis of the accumulated damage over time.

The scientific term for such data is "time series." In gear drives, a time series (time/torque/speed data)—normally measured at the input or output of the gearbox—must be in the first step of the process to get the load on a specific tooth of every gear. Then in a second step, the load spectrum (also called "duty cycle") for fatigue damage calculation must be obtained. With the load spectrum the service life of a gear can then be calculated according to the rules as described in ISO 6336-6 (Ref. 1). As main formulas for the load capacity calculation, the methods according to AGMA 2001-D4 (Ref. 3) and ISO 6336-1,2,3 (Ref. 2) can be used.

If the torque and speed in a time series is always positive, the conversion of such data in a load spectrum is carried out using a process called the "simple-count method." However, the process is more complicated when the torque and/or speed is alternating (having positive and negative values). For the other elements in a gearbox such as bearings and—with some restriction—shafts, the simple count method, extended by the consideration of speed information, can always be used to generate a load spectrum.

The flowchart (Figure 3) explains the calculation process. The time series must be filtered to obtain the load on an individual tooth (see "Extract of the load on a particular tooth from a time series"). Then, depending on if the torque is alternating or not, the simple count method can be applied (no alternating torque, see "Generation of a load spectrum with the simple count method") or the more demanding rainflow method (see "Generation of a load spectrum with the rainflow-counting algorithm") must be used.

Notes:

- The term load spectra as used in ISO 6336 (Ref. 1) or AGMA 2001 (Ref. 3) is identical to a load duration distribution (LDD) as used in the standard IEC 61400-4 (Ref. 4) for wind turbines.
- A method to calculate load spectra is not explained in AGMA rating methods (such as AGMA 2001, 2101, 2003) but a reference is given to ISO/TR 10495 (nowadays replaced by ISO 6336-6 {Ref. 1}). The procedure, based on ISO 6336-6, as discussed in this paper is applicable to AGMA ratings.

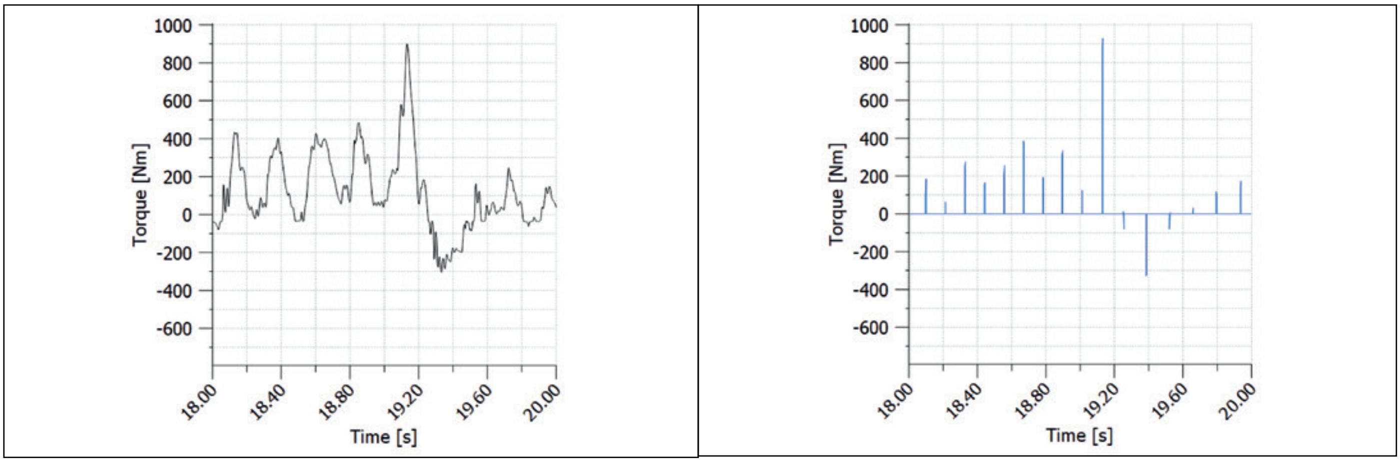

Figure 1—Torque over time in the time series (left). Torque on the tooth of the pinion at angular position 215 degrees (right).

Extract of the load on a particular tooth from a time series

The process to generate a load spectrum is complex (Figure 3). As the first step, it must be considered, that a tooth experiences load only once per revolution of the gear. The time series signal must therefore be modified before it can be used for further processing. This is shown in Figure 1.

Generation of a load spectrum with the "simple count method"

If the signs of torque and speed in a time series are in such a way that always the same flank is in contact, then for gears the so-called simple counting method can be used. Since there is only one flank in contact the respective side of the tooth root sees only pulsating tensile stress. In addition, the Hertzian pressure on the flank is always applied on the same flank. A matrix with torque intervals and speed intervals is formed and each measuring point is classified into the corresponding category, also called "bin." Then the number of measuring points is counted. This results in a load spectrum of bins with different torque and speed (extended simple count method).

The counting method is also documented in ISO 6336-6, Table 4 (Ref. 1). To obtain the load spectra for fatigue damage calculation, the range of the measured (or calculated) loads is divided into bins or classes. Each bin contains the number of load occurrences recorded in its load range. A widely-used number of bins—according to ISO 6336-6—is 64 (Figure 2). These bins can be of an equal size, but it is usually better to use larger bin sizes at the lower loads and smaller bin sizes at the upper loads in the range. In this way, the resolution for the most damaging loads is higher and the result is more accurate regarding the effective load (see "Generating the load spectra according to ISO 6336-6").

Generation of a load spectrum with the rainflow-counting algorithm

Generation of the matrix with the frequency of high to low torque

If torque and/or speed have alternating signs so that the loaded flank is changing, for the assessment of the Hertzian pressure on the considered tooth flank (left or right) only the positive values on this flank are considered. For the bending stress this simple calculation procedure cannot be applied. The considered tooth root side is subjected to an alternating load, getting tensile stress with positive torque and compression stress by negative torque. And all significant alternating load cases must be extracted from the torque curve. For that, the so-called rainflow method is used (Refs. 5, 6, 8). Rainflow analysis provides a matrix that shows how often the torque changes from T_{high} to T_{low}. The matrix, therefore, has two torque bin series, in Y-axis for T_{high} and in X-axis for T_{low} (Figure 5).

The rainflow method is usually carried out with stresses, not with torques. Because tooth root bending stress and torque are proportional, torque can also be used. In addition, the negative torques are multiplied by 1.2 since the compressive stress on the non-loaded flank is approximately 20% higher than the tensile stress on the loaded flank. Factor 1.2 is used in ISO 6336-3 and can be confirmed with FEM calculations (Fig. 4). A further challenge to get correct results: The torque must be multiplied by the dynamic factor K_V and the face load factor $K_{F\beta}$. The rainflow method does not conserve the speed information of a time series. As K_V depends on the speed, which is no longer considered in the subsequent rainflow calculation, for every data point of the time series K_V is determined and multiplied to the torque. Also, $K_{F\beta}$ must be multiplied to the torque of every point because $K_{F\beta}$ is not proportional to the torque and will therefore be different for T_{high} and T_{low}.

Conversion of rainflow T_{high} and T_{low} result to T_{ISO} and Y_M for ISO or AGMA ratings

AGMA and ISO are designed for pulsating load on the tooth; so, the nominal torque and the allowable bending stress numbers are intended for the pulsating load case. For alternate bending (reverse loading) AGMA just mentions that the allowable stress number must be multiplied by a factor 0.7. This factor coincides with older versions of ISO. In the current edition of ISO 6336-3, annex B (Ref. 2), a more precise rule is given for the alternating bending factor Y_M.

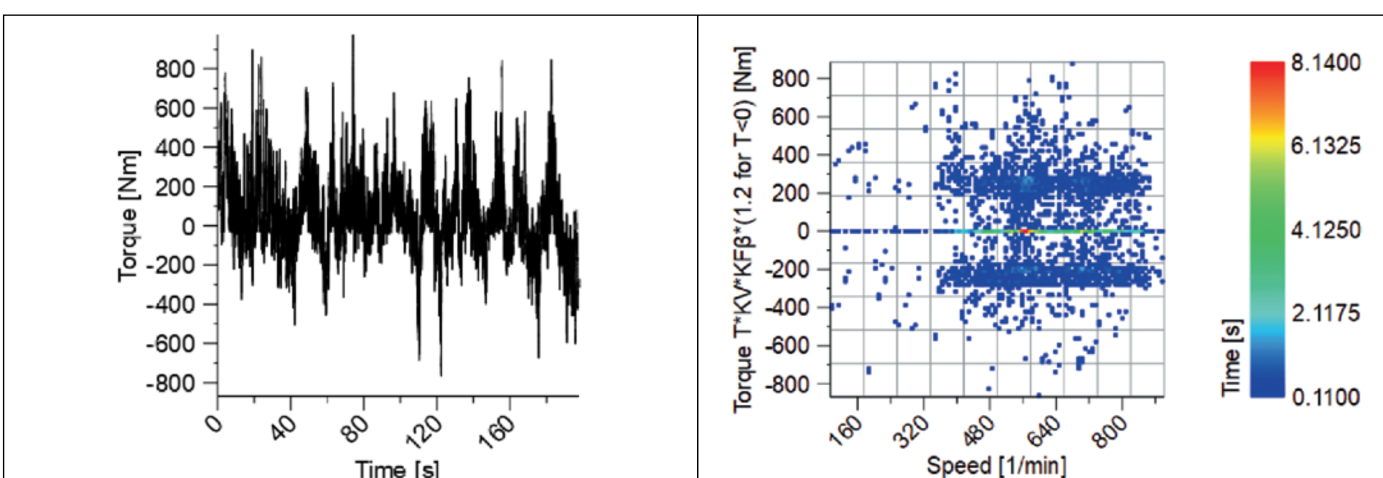

Figure 2—Torque measurement on a test run of a military car (left). Speed/Torque frequency result by simple count (right).

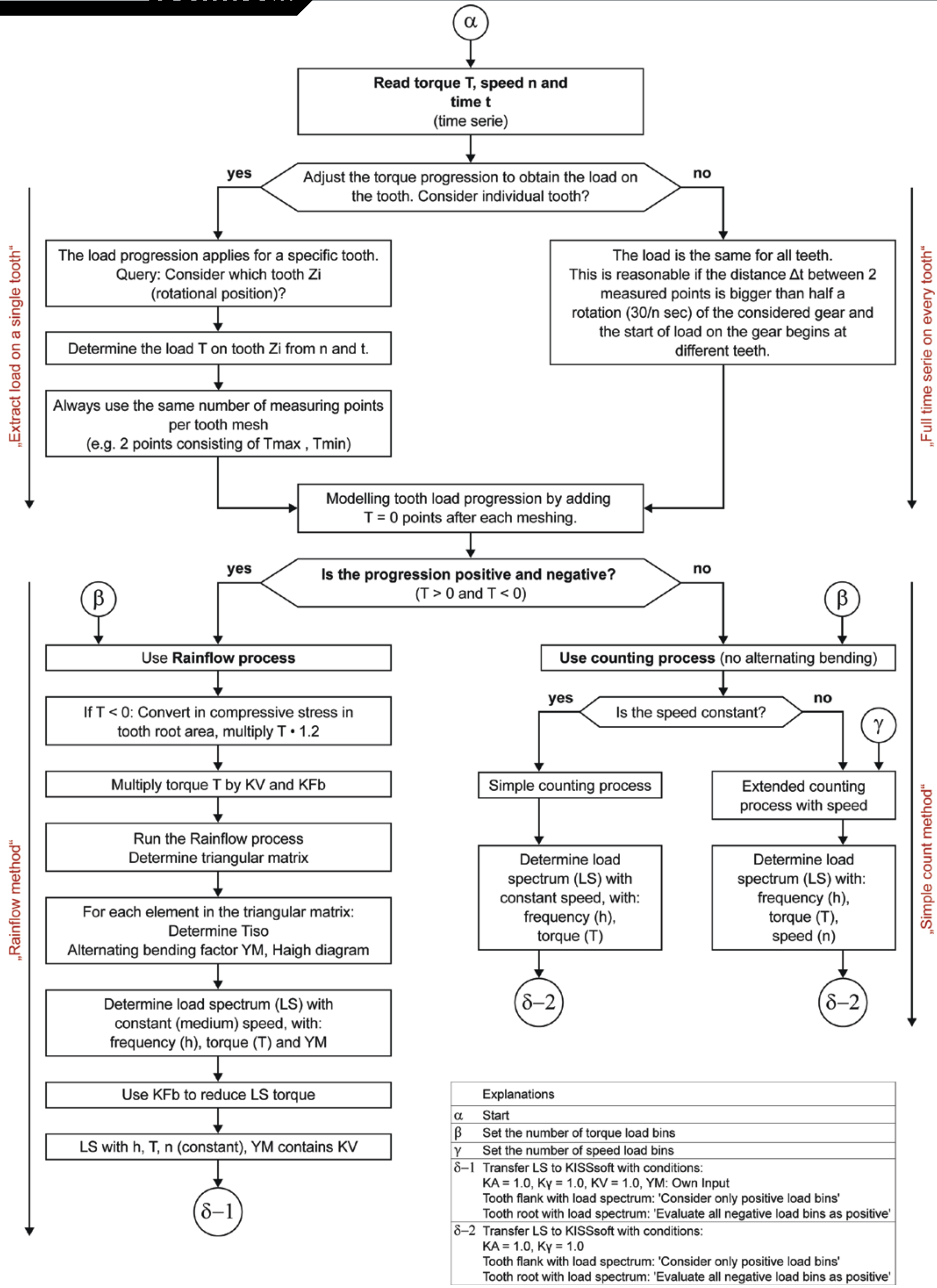

Figure 3—Flowchart to generate a load spectrum for gears from time series data.

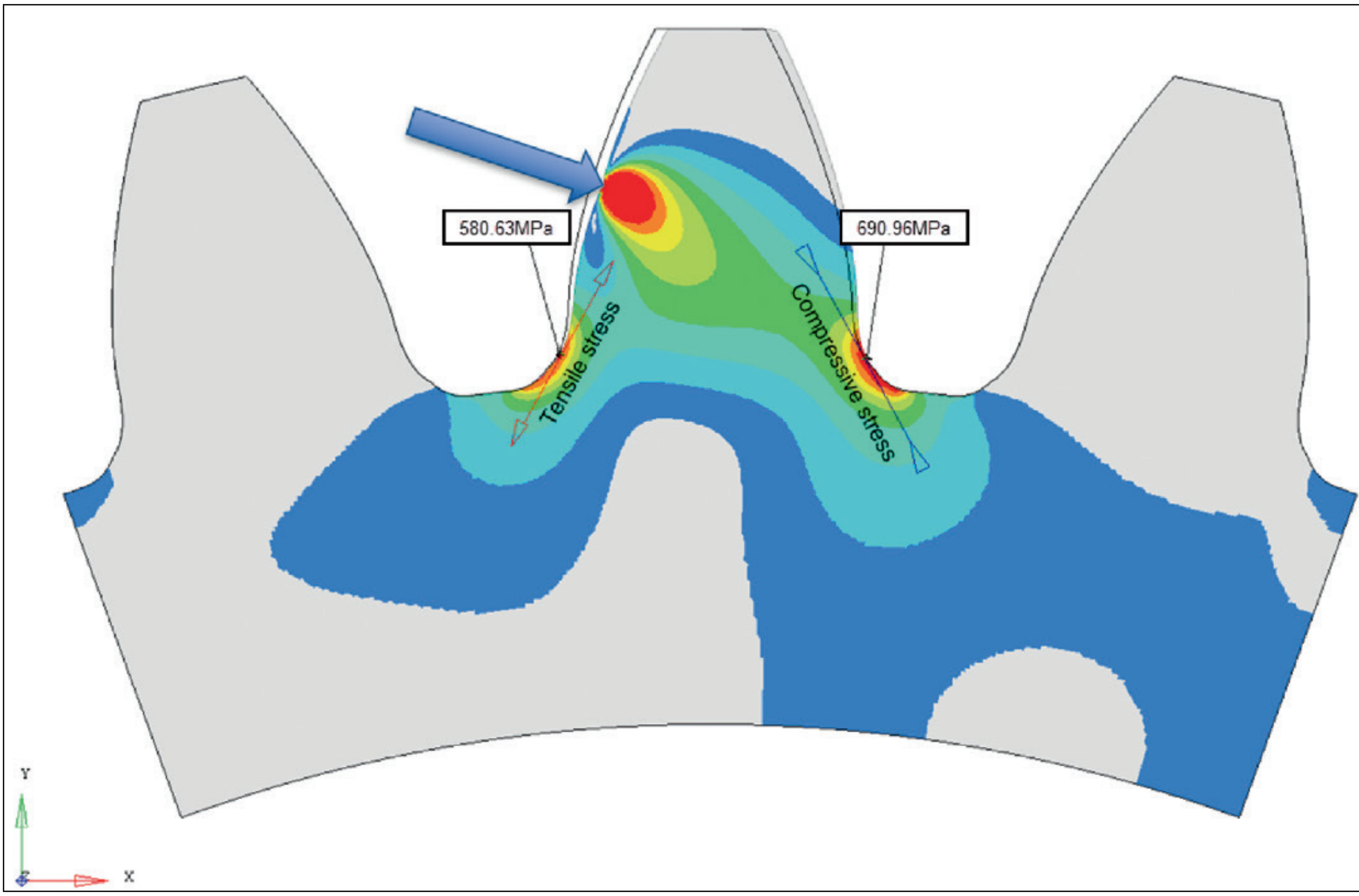

Figure 4—Stresses on the loaded and non-loaded side of a tooth by FEM analysis.

		T_{low}				
		-796.70 -778.60	-778.60 -760.49	-760.49 -742.38	-742.38 -724.28	...
T_{high}	-796.70 -778.60	01				
	-778.60 -760.49	0	2			
	-760.49 -742.38	0	0	3		
	-742.38 -724.28	01	4	0	1	
	...	...	...	...	...	...

Figure 5—Extract of a rainflow half-matrix with 100 bins.

The equation for Y_M is:

$$Y_M = \frac{1}{1 - R \cdot \frac{(1-M)}{(1+M)}} \tag{1}$$

Where:

R stress ratio. $R = -\sigma_{low} / \sigma_{high}$; and as here σ is proportional to T, $R = -T_{low} / T_{high}$

M considers the mean stress influence on the endurance strength amplitudes; the values are listed in Table 1.

Case hardened	0.8 – 0.15 YS
Case hardened and shot peened	0.4
Nitrided	0.3
Induction or flame hardened	0.4
Not surface hardened steels	0.3
Cast steels	0.4

Table 1—Mean stress ratio M according ISO 6336-3, Table B.1 (Ref. 2).

Equation 1 according to ISO may be used within a stress ratio $1 \geq R \geq 0$. Therefore Equation 1 is only valid if the following conditions apply: $T_{high} > 0$; $T_{low} < 0$ and $-T_{low} \leq T_{high}$. As the general case R may be in the range from -∞ … to +1, Equation 1 must be extended. For the definition of the allowable stress in mechanics the Haigh diagram is appropriate (Figure 6). The construction of the Haigh diagram requires the tensile strength R_m, the yield strength R_{p02}, the tooth root fatigue strength for pulsating loads (sat according to AGMA, or σ_{Flim} according to ISO 6336) and the mean stress ratio M.

Basically, the alternating bending factor Y_M is a factor that considers the change in the admitted amplitude σ_{admAmp} and the occurring amplitude $\sigma_{LoadAmp}$ of the general case "Gen" compared to the pulsating load case "Puls."

$$\frac{\sigma_{admAmpPuls}}{\sigma_{LoadAmpPuls}} = Y_M \cdot \frac{\sigma_{admAmpGen}}{\sigma_{LoadAmp\ Gen}} \tag{2}$$

The permissible amplitude $\sigma_{admAmpGen}$ results from the intersection of the R_{Gen} line with the Haigh diagram. With the high stress σ_{high}, the amplitude $\sigma_{LoadAmpGen}$ results in the general case from:

$$\sigma_{LoadAmpGen} = \frac{(\sigma_{high} - \sigma_{low})}{2} = \frac{(\sigma_{high} - R_{Gen} \cdot \sigma_{high})}{2} = (1 - R_{Gen}) \cdot \frac{\sigma_{high}}{2} \tag{3}$$

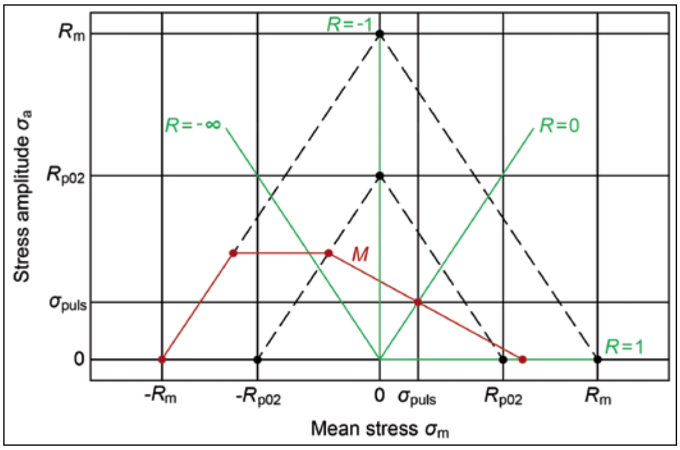

Figure 6—Haigh diagram with mean stress influence M according ISO 6336-3. (Note: The red line "M" is defined by the point {$\sigma_{admAmpPuls}$, $\sigma_{admAmpPuls}$} and gradient M.)

For the pulsating case "Puls" with $R_{Puls} = 0$ the following applies:

$$\sigma_{LoadAmpPuls} = \frac{(\sigma_{high} - \sigma_{low})}{2} = \frac{(\sigma_{high} - R_{Puls} \cdot \sigma_{max})}{2} = \frac{\sigma_{high}}{2} \tag{4}$$

Whereas the admitted stress for pulsating bending is:

$$\sigma_{admAmpPuls} = s_{at}/2 \tag{5}$$

Finally, when Equations 3, 4 and 5 are inserted in Equation 2; then rearranged, we get Y_M for the general case:

$$Y_M = \frac{2}{s_{at}} \cdot \frac{\sigma_{admAmpGen}}{(1 - R_{Gen})} \tag{6}$$

Where:

s_{at} Allowable bending stress number (bending) in AGMA 2101 (Ref. 3)

$\sigma_{admAmpGen}$ The permissible amplitude from Haigh diagram

It must be noted, that with this general definition of Y_M all load cases on the tooth root can be considered; also pulsating compressive stress. In such cases, Y_M will be bigger than one, because the admitted amplitudes in the compressive domain of the Haigh diagram are bigger than the amplitudes in the tension domain.

T_{max}	T_{min}	$T_{ISOorAGMA}$
Positive	Positive	$+T_{max}$
Positive	Zero	$+T_{max}$
Positive	Negative	$+T_{max}$
Zero	Negative	$-T_{min}$
Negative	Negative	$-T_{min}$

Table 2—Determination of the nominal torque to be used as nominal load in AGMA or ISO.

Generating the load spectra according to ISO 6336-6

With the nominal torque from Table 2 and the alternating bending factor Y_M from Equation 6, every element of the rainflow matrix can now be converted in a bin of the load spectrum. A bin according to ISO 6336-6 (Ref. 1) consists of only two elements, frequency, and torque. The frequency is a percent value of the number of load cycles of the bin versus the total number of cycles. The torque is the $T_{ISO\ or\ AGMA}$ value according to Table 2. For more general use, this definition must be extended. Generally, it is good practice to add speed as the third element of the bin. This definition of a bin (frequency, torque, and speed) is the most used case for gear and bearing calculations, appropriate to the simple-count method (see "Generation of a load spectrum with the simple count method"). When the rainflow method is used, then for every bin in addition the values Y_M, one for every gear, must be added (Figure 7).

Application of load spectra in AGMA ratings

A method to calculate load spectra is not explained in AGMA rating methods (such as AGMA 2001, 2101, 2003) but a reference is given to use Miner's rule as presented in ISO/TR 10495. This norm was replaced in 2006 by ISO 6336-6; the current version is from 2019 (Ref. 1). Gear rating methods according to AGMA and ratings according to ISO have the same basic structure, using general factors and bending and pitting ratings. Both methods use stress cycle factors (as called in AGMA) or

Calculation with load spectrum — No.

	Frequency [%]	Torque [Nm]	Speed [1/min]
70	0.222000	852.2800	588.7200
71	0.032000	852.2800	751.0800
72	0.063000	929.7600	426.3600
73	0.158000	929.7600	588.7200
74	0.032000	929.7600	751.0800
75	0.032000	1007.2400	588.7200
76	0.063000	1162.2000	588.7200

Calculation with load spectrum — No. 1 — Own Input

	Frequency [%]	Torque [Nm]	Speed [1/min]	$K_{H\beta}$	Y_{M1}	Y_{M2}
57	0.069000	861.2500	479.1600	1.0381	0.8912	0.8928
58	0.069000	861.2500	479.1600	1.0381	1.0000	1.0000
59	0.069000	949.6500	479.1600	1.0346	0.8573	0.8593
60	0.139000	949.6500	479.1600	1.0346	1.0000	1.0000
61	0.069000	1037.8550	479.1600	1.0316	0.7516	0.7547
62	0.069000	1037.8550	479.1600	1.0316	0.7814	0.7843
63	0.069000	1037.8550	479.1600	1.0316	1.0000	1.0000

Figure 7—Load spectra examples. Typical simple count method format (left) with frequency, torque, and speed. Typical rainflow method format (right) includes alternating bending factors Y_M.

life factors (as called in ISO) representing the SN-curves of gear materials. Otherwise, the factors to calculate the stress values and the allowable stresses are sometimes significantly different. The calculation of service life under variable load, as prescribed by ISO 6336-6 (Ref. 1), is using the Palmgren Miner rule, which is a widely used linear damage accumulation method. The method is absolutely "neutral," which means, that individual factors of the rating method used, are not involved. Therefore, the combination of the ISO 6336-6 method with an AGMA rating is possible and well suitable.

The alternate bending factor Y_M, as discussed in "Conversion of Rainflow T_{high} and T_{low} result to T_{ISO} and Y_M for ISO or AGMA ratings," is only marginally treated in AGMA, only "Use 70 percent of the sat values for … gears where the teeth are completely reverse loaded on every cycle" is mentioned in AGMA 2001, clause 16.2. As this factor in ISO 6336-3 is based on the Haigh diagram, which is a widely accepted concept in material mechanics, it can be assumed, that the factor is also applicable for AGMA ratings.

Gear rating calculation example

A load spectrum can be treated in a gear software package. Usually, a spectrum is introduced manually bin by bin or may be read in from an Excel file. If the spectrum is generated from a time series some additional inputs are needed (Figure 8).

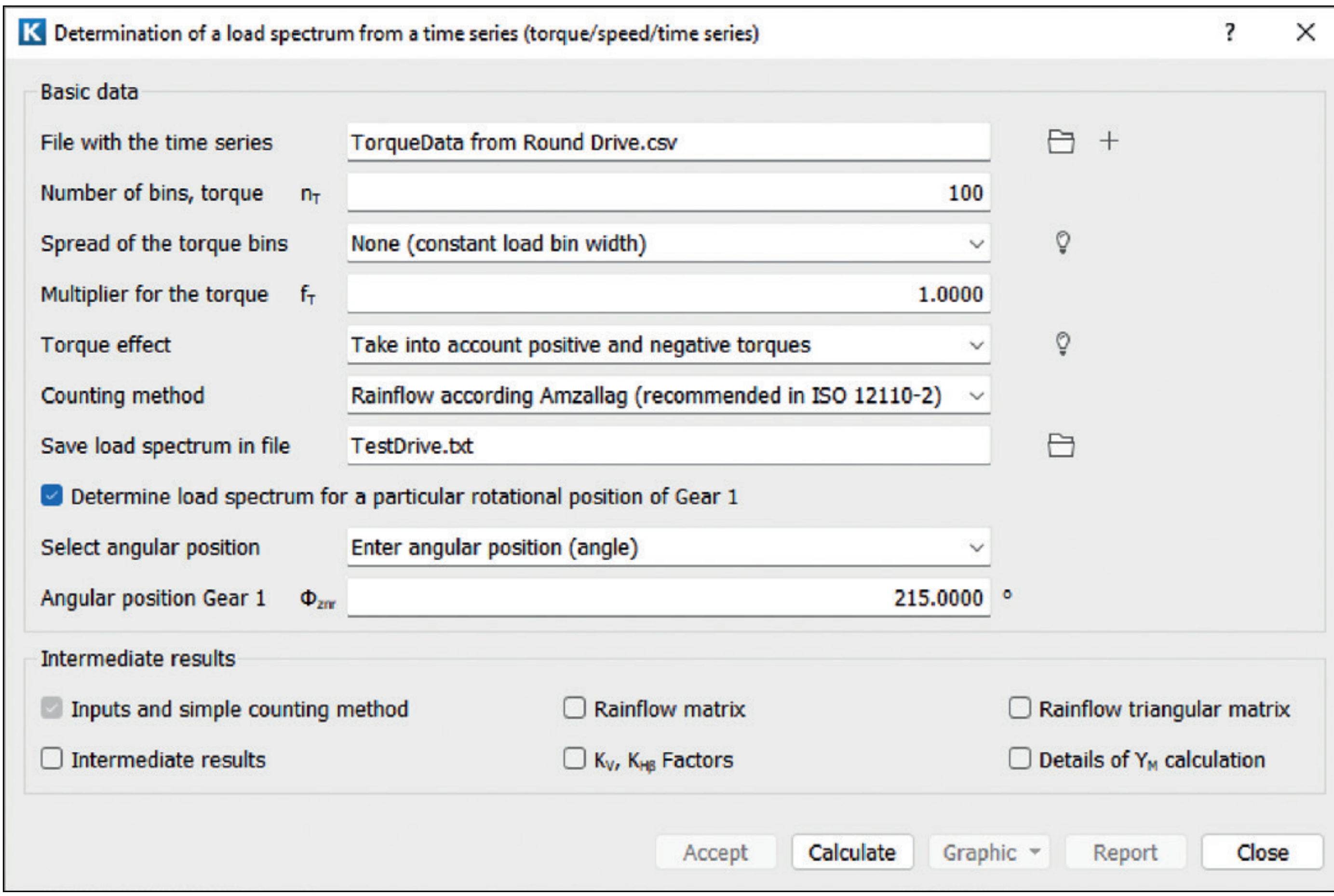

Figure 8—Input window in KISSsoft (Ref. 7) for a duty cycle determination from a time series.

	Tooth No. 5 Maximum damage		Tooth No. 1 Mean damage		Tooth No. 17 Minimum damage	
Safety Bending SF1, SF2	1.441	1.455	1.472	1.491	1.525	1.540
Safety Pitting SH1, SH2	1.173	1.277	1.179	1.283	1.189	1.254
Damage bending of gear 1, gear 2 (%)	86.47	78.99	71.63	64.17	49.60	46.36
Damage pitting of gear 1, gear 2 (%)	11.27	3.79	9-23	3.36	9.41	3.17
Lifetime of gear 1, gear 2 (h)	6938	7596	8376	9351	12100	12940

Table 3—Safety, damage, and lifetime, calculated with the time series on different teeth.

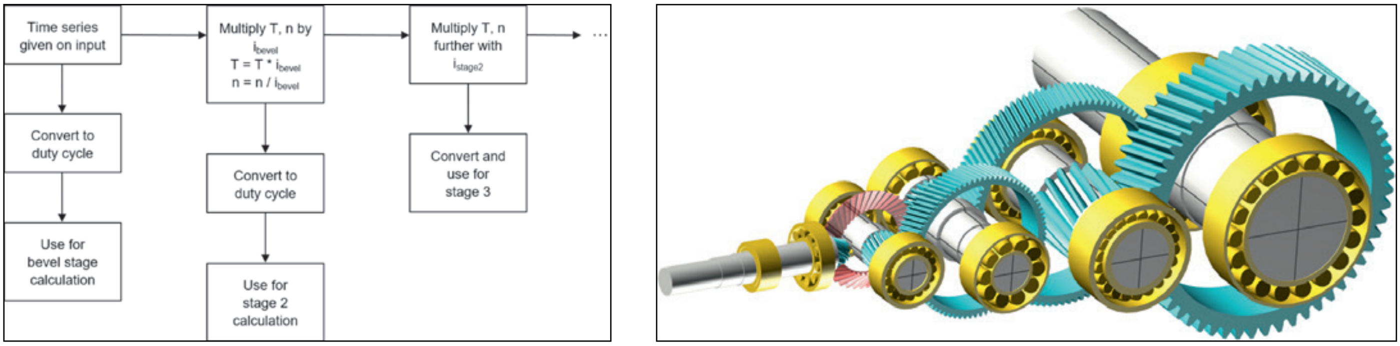

Figure 9—Industrial gearbox in KISSsys *(Ref. 7), flow chart for the load spectrum determination.*

One of the interesting results is the dependency of the considered tooth. As the torque in the time series in Figure 3 is strongly varying, the extract of the load on a particular tooth (Figure 1) gives different safety or lifetime results, see Table 3. But in this example, the duration of the time series is only 190 seconds (short test run). The test run will be repeated several times, the tooth in contact, when starting, will always be different—so the tooth getting the biggest damage will change. Therefore, it is a better practice to consider the results on a tooth having mean damage.

Application of time series in drive train calculations

Today the analysis of drive trains is performed with appropriate software as *KISSsys* (Ref. 7), which models the complete drive with all the main elements such as gears, shafts, and bearings. Normally the time series is given for the input or the output coupling. A load spectrum can be defined at this position and being used all over the drive system. A problem arises when time series with positive and negative torques are used. Then, because of the different frequency of alternating load changes, for every gear stage, an appropriate load spectrum must be used (Figure 9).

To motivate this statement, suppose that a tooth on the pinion of the input stage is submitted during 60 rotations to a positive torque, then for additional 60 rotations to a negative torque, and so on. If the input stage reduction is i = 4.0, then the pinion of Stage 2 will rotate four times lower; therefore, getting a torque change after 15 rotations. So, the frequency of alternate bending cycles is four times higher on Stage 2, and so on for the next stages.

Conclusion

AGMA and ISO gear ratings can be executed with load spectra (duty cycles)—based on Miner's rule—as explained in ISO 6336-6 (Ref. 1). Load spectra may be defined by different methods. In this paper, the ability to generate a duty cycle from a time series (time-torque-speed data) is explained. Such a time series can be obtained by measurements of the operating loads on an application or by computer simulation.

If the torque and speed in such a time series are so, where the same flank is always in contact, a simple-count method can be used to generate the duty cycle. However, if the loaded flank is alternating, the considered tooth root is not only submitted to pulsating tensile stress (bending) but additionally to alternating bending and pulsating compressive stress. As the first step, all the significant torque changes must be extracted from the time series. For this, the rainflow method is used. As the second step, the result of rainflow must be converted into a definition fitting AGMA or ISO gear ratings. This means that for a bin of the load spectrum the nominal torque and the alternate bending factor must be determined.

Finally, the application of the method in gear calculations and in drive train analysis is discussed with an example.

References

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Dr. Ulrich Kissling was born in Zurich. He studied Machine Engineering at the Swiss Technical University (ETH). He continued his academic career with a doctorate. In 1981, he started his professional career as calculation engineer in a Gearbox Manufacturing Company in Zurich, continued then as Technical Manager and Managing Director.

As calculation engineer for gearbox design, he started to develop software for gear, bearing and shaft layout. In 1985, he named this software *KISSsoft* and started to market it. In 1986 the first license was sold. In 1998, he founded his own company, KISSsoft AG, to take care of the software activities. Since then, the staff of KISSsoft AG is growing constantly from three people in 1998 to over 40 in 2022. Today the software is the leading drive train design software, used by more than 3500 companies on all continents.

As a gear expert Dr. Kissling is actively participating in different Work Groups of ISO for the development of international standards.